

# How Arbitrage-Free is the Nelson-Siegel Model under Stochastic Volatility?

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This study examines the effect of no-arbitrage on the Nelson-Siegel (NS) (1987) model under stochastic interest-rate volatility. Unlike under constant volatility, in which only a constant (convexity-adjustment) term of a yield function differs with and without no-arbitrage, the loadings of the factors driving the yield curve also differ when the volatility is spanned by the factors even after controlling for the drift of changes in the factors. We specify the volatility as a linear function of the factors while keeping the other components of the model unchanged, which allows for interpreting the spanned-volatility model as an extension of the NS model. We consider two approaches for examining the no-arbitrage effect. The first is the two-step approach in which the factors are first extracted by fitting the NS curve at each point in time and then the dynamics of the factors are estimated from the pooled data. These factor and parameter values are fed into the no-arbitrage version of the models to compute the no-arbitrage yield curve, which is compared to the original NS curve. The second is the one-step approach in which the factor and parameter values are estimated simultaneously in both cases with and without no-arbitrage and then compared between the two cases. We find in both approaches that spanned volatility does not magnify the effect of no-arbitrage relative to constant volatility on average, though it does when the level of interest rates is very high. The finding implies that we can pursue practical benefits of the NS model, such as by augmenting it with stochastic volatility after identifying the factors from the yield curve, without much increasing the cost of the theoretical deficiency of allowing arbitrage opportunities.